INTERNATIONAL UNIVERSITY OF EAST AFRICA

'Learning to Succeed'

FACULTY OF ENGINEERING

END OF SEMESTER EXAMINATIONS - APRIL 2025

PROGRAMME: BACHELOR OF CIVIL ENGINEERING

YEAR/SEM: YEAR 2/SEMESTER 1

COURSE CODE: EMT2123

NAME: ENGINEERING MATHEMATICS III

DATE: 2025-04-16

TIME: 2:00-5:00PM

INSTRUCTIONS TO CANDIDATES:

1. Read the instructions very carefully
2. The time allowed for this examination is STRICTLY three hours
3. Read each question carefully before you attempt and allocate your time equally between all the Sections
4. Write clearly and legibly. Illegible handwriting cannot be marked
5. Number the questions you have attempted
6. Use of appropriate workplace examples to illustrate your answers will earn you bonus marks
7. Any examination malpractice detected will lead to automatic disqualification.

DO NOT WRITE ANYTHING ON THE QUESTION PAPER

Section A Attempt 2 questions from this section.

Question 1:

5. (a) (i) Show that $\mathcal{L}\ [c_1 f(t) + c_2 f(t)] = c_1 \mathcal{L} f(t) + c_2 \mathcal{L} g(t)$ (5 marks)

(ii) $\mathcal{L} f'(t) = s\mathcal{L} f(t) - f(0)$ (5 marks)

(b) Find the Laplace transformation of

$$f(t) = \begin{cases} 1 & 0 \le t \le 2 \\ t-2 & t \ge 2 \end{cases}$$ (15marks)

Question 2:

4. Find the general solutions to the following differential equations

$$\frac{dx}{dt} = -4x + y + z,\ \frac{dy}{dt} = x + 5y - z,\ \frac{dz}{dt} = y - 3z$$

(25 marks)

Question 3:

1. **(a)** Given the system of differential equations

$$\frac{dy}{dt} = -6x + 3y$$
$$\frac{ay}{dt} = 4x + 5y.$$

(i) Express the equations in the form $x' = Ax$

(ii) Find the Eigen values of A

(iii) Find the Eigen vectors corresponding to the Eigen values. Hence write the solution to the differential equations. **(25 marks)**

Question 4:

2. **(a)** Use the ***Laplace transforms table*** given below to evaluate the following.

$\ell[\,1] = \frac{1}{s}$	$\ell[e^{at}] = \frac{1}{s-\alpha}$
$\ell[\,t] = \frac{1}{s^2}$	$\ell[te^{-at}] = \frac{1}{(s+\alpha)^2}$
$\ell[\,t^2] = \frac{2}{s^3}$	$\ell[te^{at}] = \frac{1}{(s-\alpha)^2}$
$\ell[\,t^3] = \frac{6}{s^4}$	$\ell[t^n e^{-at}] = \frac{n!}{(s+\alpha)^{n+1}}$
$\ell[\,t^n] = \frac{n!}{s^{n+1}}$	$\ell[t^n e^{-at}] = \frac{n!}{(s+\alpha)^{n+1}}$
$\ell[t^3] = \frac{6}{S^4}$	$\ell[\cos\omega t] = \frac{s}{s^2+\omega^2}$
$\ell[t^n] = \frac{n!}{S^{n+1}}$	$\ell[\sin\omega t] = \frac{\omega}{s^2+\omega^2}$
$\ell[e^{-at}] = \frac{1}{s+\alpha}$	$\ell[e^{-at}\sin\omega t] = \frac{\omega}{(s+\alpha)^2+\omega^2}$
	$\ell[e^{-at}\cos\omega t] = \frac{\omega+\alpha}{(s+\alpha)^2+\omega^2}$

i) $\ell[t^5]$

ii) $\ell[te^{-6t}]$

iii) $\ell[te^{3t}]$

iv) $\ell[\sin 5t]$

v) $\ell[e^{-2t}\sin 5t]$ **(15 marks)**

(b) By integration, find the Laplace transform of $f(t) = t^2$. **(10 marks)**

Section B Attempt 2 questions from this section.

Question 1:

9. The population x of Kampala engineers follows a logistic model

$$\frac{dx}{dt} = \frac{1}{100}x - \frac{1}{10^8}x^2$$

Where t is the time in years. Given that the population was 100000 in 1990

(i) Determine the population as a function of time t.

(ii) In what year will the population double

(iii) How large will the population ultimately be? **(25 marks)**

Question 2:

8. Solve the following second order differential equation. The initial condition applies to all the parts of the differential equations

(i) $y'' - 6y' + 5 = 0$ **(5 marks)**

(ii) $y'' - 6y' + 9y = 0$ **(10 marks)**

(iii) $y'' + 11y' + 24y = 0$ **(10 marks)**

$y(0) = 0 \quad y'(0) = -7$

Question 3:

11. Find the first and second order partial derivative of the following functions.

(a) $z = x^3 + xy + y^2 + 5x - 5y + 3$ **(10 marks)**

(b) $x^3 - 6xy + y^3$ **(5 marks)**

(c) $10xy - 3x^2 - 4y^2 - 2x - y + 5$ **(10 marks)**

Question 4:

7. Solve the following differential equation

(a) $x^1 + (\tan y)x = \cos^2 y$ **(10 marks)**

(b) $\cos^2 x \sin x \dfrac{dy}{dx} + (\cos^3 x)y = 1$ **(15 marks)**